# The Diet Problem

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This is a story about connections. If a certain event hadn't happened way back in 1937, then 10 years later it is certain that linear programming and the simplex method would never have happened (at least not then), and many people's lives and the way some enterprises plan their future would have turned out quite differently.

If Milton Friedman hadn't left the Bureau of Labor Statistics in Washington to go to the University of Chicago back in 1937, then there wouldn't have been a job vacancy and George Dantzig wouldn't have been hired by the bureau's Urban Study of Consumer Purchases as a CAF-2 Statistical Clerk at $1,440 per year. Jobs were hard to come by in the days of the Great Depression.

At the Urban Study, I became good friends with Duane Evans, who during the war years 1941–45 teamed up with Jerry Cornfield and Marvin Hoffenberg to collaborate with Wassily Leontief on the development of the input–output model of the national economy. Linear programming had its origins in the I/O model.

During the war, I was in charge of Airforce Statistical Control's Combat Analysis Branch in the Pentagon. I was much too busy during the day to see Duane, or Jerry, or Marvin, but at night over the phone I learned from Duane about the remarkable mathematical properties of the input-output matrix. And so when, in 1946–47, an opportunity came my way to mechanize the airforce's planning process, the first thing I thought of using was the I/O model. It had to be generalized, of course, to make it dynamic and have alternative activities, and it had to be given an


0091-2102/90/2004/0043$01.25

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INTERFACES 20: 4 July–August 1990 (pp. 43–47)

objective function as a way to drive the system towards optimality.

Having formulated the linear programming model, I next had to find a way to solve it. First I searched the literature, for it seemed to me that such an obviously useful model must have been much studied. I sought the help of others like Tjalling Koopmans (who later received the Nobel Prize in Economics for his work in the field) and the help of Johnny von Neumann (the world's leading mathematician, who stated but did not formally prove the famous duality theorem). Having had no luck with my search for an algorithm, I proposed one of my own invention.

The next step was to find a good problem to test the newly proposed simplex method. At a little bull session at the Pentagon with the Bureau of Labor's input/output team, Marvin Hoffenberg suggested we test it on Jerry Cornfield's diet problem. Jerry said that he had worked on the problem several years earlier for the Army who wanted a low cost diet that would meet the nutritional needs of a GI soldier.

It turned out that Cornfield couldn't find his input data and that it was necessary to reconstruct the diet problem from scratch. I followed up three leads. The cost of foods I could get from the Bureau of Labor Statistics. The nutritional requirements of a person could be obtained from a brochure issued by the National Research Council of the National Academy of Sciences. For the nutritional contents of various foods, I contacted the Bureau of Home Economics of the Department of Agriculture.

I will never forget my conversation with the home economist, a well-known authority on nutrition. She was patient, warm, and cooperative until she understood exactly what it was I wanted. Suddenly I could feel icicles forming as she spoke. She said to me: "I suggest you read the article by George Stigler on . . . da . . . da and after that read . . . da and . . . da." She spoke so rapidly that I had to ask her to repeat the references. She did, said goodby, and showed me the door. Naturally I became curious. I wondered what it was in those references, or about the diet problem that could possibly provoke such a reaction.

## You could feel the icicles forming as she spoke.

George Stigler's article on "The cost of subsistence" [1945] turned out to be quite remarkable. Indeed I was so impressed with it that I included a paraphrased version of it as Chapter 27 in my book *Linear Programming and Extensions* [Dantzig 1963]; the title of the chapter is "Stigler's nutrition model: An example of formulation and solution." Stigler (who later received the Nobel Prize) had a reputation in those days as an acid critic who tore the research of his fellow economists to pieces. During the war, Stigler found himself working on the same diet problem as Jerry Cornfield.

Could it be that Stigler was in the awkward position of having to be constructive and knew he was vulnerable? To protect himself from criticism, was he going out of his way to explain in detail all the weaknesses of his model? The paper contains a delightful passage on the question of when is an apple an apple and what do you mean by its cost and nutritional content? For example when you say *apple* do you mean a Jonathan, or McIntosh, or Northern Spy, or Ontario, or Winesap, or Winter Banana? You see, it can make a difference, for the amounts of ascorbic acid (vitamin C) can vary from 2.0 to 20.8 units per 100 grams depending upon the type of apple. It also makes a difference whether the apple is freshly picked or has been stored in a barrel for six months. And then there is the question of whether you are eating it raw or in a pie.

In spite of all the ambiguities, Stigler finally settled on a model and, since there was no technique that he was aware of to solve it, he invented a very clever heuristic to arrive at a diet that cost only $39.93 per year (1939 prices). He did not claim it to be the cheapest solution but gave good reasons for believing that the annual cost could not be reduced much.

## When you say *apple* do you mean a Jonathan, or McIntosh, or Northern Spy, or Ontario?

What had infuriated the home economist was a footnote at the very end of Stigler's article to the effect that tax-supported bureaucrats (and others of similar ilk) for reasons best known to themselves had recommended low cost diets that cost twice as much.

In the fall of 1947, Jack Laderman of the Mathematical Tables Project of the National Bureau of Standards undertook as a test of the newly proposed simplex method the determination of a least cost adequate diet based on Stigler's data. It was the first "large scale" computation in the field. The system consisted of 9 equations in 77 unknowns. Jack parcelled out a different 8 or 9 columns (of the 77 columns) to each of the nine clerks who were assigned to process them. Using hand-operated desk calculators (this was in the days before computers), the nine clerks took approximately 120 man days to obtain an optimal solution of $39.69. Stigler's heuristic solution was only off from the true annual optimal cost by 24 cents: not bad!

The clerks recorded their work for each iteration on a separate work sheet. Later, after the job was complete, Jack joined the separate sheets together to form one large sheet which we dubbed the Table Cloth. I have a letter in my files from Oskar Morgenstern (who with von Neumann wrote the famous treatise on game theory) saying he would like to come down from Princeton to Washington to view it. Eventually the table cloth disappeared, never to be seen again.

But this is not the end of the diet tale. In the early 1950s, I moved to Santa Monica to work for the RAND Corporation. My doctor advised me to go on a diet to lose weight. I decided I would model my diet problem as a linear program and let the computer decide my diet. Some revisions of the earlier model, of course, would be necessary in order to give me a greater variety of foods to choose from; the calorie intake had to be reduced to under 1,500 calories per day; and the objective function had to be changed (I wasn't interested in saving money). I said to myself: "The trouble with a diet is that one's always hungry. What I need to do is maximize the feeling of feeling full." After giving much thought to what the coefficients in the objective form should be, I used the weight (per unit amount) of a food minus the weight of its water content. Input data for over 500 different foods was punched into cards and fed into Rand's IBM 701 computer.

## I decided I would let the diet model program my diet.

My colleague Ray Fulkerson (famous for his contributions to network flow and matroid theory) was skeptical. "You crazy or something? We solve models to obtain optimal schedules of activities for others to follow, not for ourselves." Nevertheless I was determined to do just that.

One day I said to Anne, my wife, "Today is Der Tag, whatever the 701 says that's what I want you to feed me each day starting with supper tonight."

Around 5:00 PM, Anne called, "Nu, it's five and you haven't called. What should I be cooking?" I replied that she didn't really want to know. I then read off the amounts of foods in the optimal diet. Her reaction: "The diet is a bit weird but conceivable. Is that it?"

"Not exactly," I replied, "AND 500 gallons of vinegar." She thought it funny and laughed.

I figured there had to be a mistake somewhere. It turned out that our data source listed vinegar as a very weak acid with water content = zero. Therefore, according to the way the model was formulated the more vinegar you drank the greater would be your feeling of feeling full. I decided that vinegar wasn't a food.

The next day the above scene was repeated except this time I called Anne in time to prepare supper. Again the diet seemed to be plausable except for calling for the consumption of 200 bouillon cubes per day. Anne made one of the great puns of all time: "What are you trying to do, corner the bullion market?" The next day started with a test to see how many Bovril bouillon cubes I could consume for breakfast. I decided to begin by mixing four in a cup of hot water. I had to spit it out: it was pure brine! I called my doctor and asked him how come the nutritional requirements didn't show a limit on the amount of salt? "Isn't too much salt dangerous?" He replied that it wasn't necessary; most people had enough sense not to consume too much. I placed an upper bound of three on the number of bouillon cubes consumed per day. That was how upper bounds on variables in linear programming first began.

The next day the above scene was repeated, except this time the diet called, among other things, for two pounds of bran per day. Anne said, "If you consume that much bran, I doubt you'll make it to the hospital. I'll tell you what I will do (she was beginning to take charge): I'll buy some finely milled bran and limit you to no more than a couple of cupfuls per day." The model was revised with an upper bound put on the amount of bran. The next day the proposed menu was almost exactly the same except this time it was two pounds of blackstrap molasses which substituted for the bran; apparently their nutritional contents were quite similar.

At this point Anne got tired of the whole game. Speaking firmly so that I would know who was boss, she said, "I have been studying the various menus the computer has been generating. There are some good ideas there that I can use. I'll put you on MY diet. She did and I lost 22 pounds.

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